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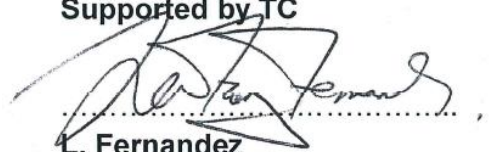


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## 1. INTRODUCTION

Life Data Analysis refers to the application of statistical analyses to determine the reliability characteristics of either components or systems based on actual failure data. The required failure data may originate from component or system development (e.g. test data), or may be recorded during operations and maintenance using an appropriate FRACAS (Failure Reporting and Corrective Action System).

It is imperative to distinguish between component Life Data Analysis (i.e. non-repairable items where data typically consists of individual times-to-failure) and system Life Data Analysis (i.e. repairable systems where data typically consists of times between successive failures in a single system).

## 2. SUPPORTING CLAUSES

### 2.1 SCOPE

This guideline describes the process of performing Life Data Analysis on actual failure data. It briefly refers to both component and system Life Data Analysis and includes a few examples to illustrate the differences between them. Since it is not possible to include comprehensive information in this guideline, the user should consult with other references for a more detailed discussion.

#### 2.1.1 Purpose

The purpose of this document is to provide guidance on performing failure analysis of either component or system failure data.

#### 2.1.2 Applicability

This document shall apply throughout Eskom Holdings Limited Divisions. The intended users of this guideline include both Eskom technical personnel and sub-contractors. It is applicable primarily during operations and maintenance but can also be used during system design (e.g. analysis of test failure data).

### 2.2 NORMATIVE/INFORMATIVE REFERENCES

Parties using this document shall apply the most recent edition of the documents listed in the following paragraphs:

#### 2.2.1 Normative

- [1] ISO 9001, *Quality Management Systems*

#### 2.2.2 Informative

- [2] P.D.T. O'Connor and A. Kleyner, *Practical Reliability Engineering*, 5<sup>th</sup> edition, John Wiley, 2012
- [3] C.E. Ebeling, *An introduction to Reliability and Maintainability Engineering*, 2<sup>nd</sup> edition, Waveland Press, 2010
- [4] [www.weibull.com](http://www.weibull.com)

### 2.3 DEFINITIONS

NONE

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### 2.3.1 Disclosure Classification

**Controlled Disclosure:** Controlled Disclosure to external parties (either enforced by law, or discretionary).

## 2.4 ABBREVIATIONS

Abbreviation	Description
$f(t)$	Probability density function (pdf)
$F(t)$	Cumulative distribution function (cdf)
FRACAS	Failure Reporting and Corrective Action System
IID	Independently and Identically Distributed
HPP	Homogenous Poisson Process
MTBF	Mean Time Between Failure
MTTF	Mean Time To Failure
NHPP	Non-homogenous Poisson Process
$R(t)$	Reliability function
$\beta$	Shape parameter
$\lambda(t)$	Hazard (or failure) rate
$\eta$	Characteristic life, or scale parameter
$\gamma$	Failure-free life, or location parameter

## 2.5 ROLES AND RESPONSIBILITIES

None

## 2.6 PROCESS FOR MONITORING

None

## 2.7 RELATED/SUPPORTING DOCUMENTS

None

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### 3. LIFE DATA ANALYSIS OVERVIEW

LDA may be divided into two groups, namely *Component Life Data Analysis* and *System Life Data Analysis*. Component Life Data Analysis, which requires individual time-to-first-failure of non-repairable items, is typically analysed using Weibull analysis. The identified failure distribution (with associated distribution parameters) may be useful to influence maintenance strategies e.g. an item subject to wear-out may benefit from preventive maintenance. System Life Data Analysis refers to the analysis of failures occurring in repairable systems, which, as an example of a series of discrete events, can be analysed using event series analysis.

Both Component LDA and System LDA are briefly discussed in this guideline and it is imperative to understand the differences between them. Unless the available failure data is carefully examined and correctly analysed, the analysis can lead to incorrect and misleading results. Care should especially be taken when analysing failure data for repairable systems collected automatically by failure reporting systems.

Knowledge of the pattern of failure (i.e. failure distribution) is important to understand the dominant failure mechanisms, which, in turn, is necessary to improve component (and system) reliability. Improvement in reliability can be achieved by implementing a change in 1) design (e.g. redesign or modification), or 2) operations (e.g. duty cycle), or 3) maintenance (e.g. preventive maintenance).

Regardless of the specific analysis requirement, it is recommended that at least a basic Pareto Analysis (i.e. “the significant few and the insignificant many” principle) be performed on available data. It is often found that a large proportion of failures are due to a small number of causes. Therefore, Pareto Analysis may be useful to plan further detailed LDA, where required.

LDA is typically performed using an applicable software application but analysis can also be performed using graphical methods, i.e. probability plotting.

In practice, LDA may be continually updated as more failure data becomes available. The results of LDA should, therefore, be put under configuration management (inclusive of identification of source of data, raw data used, software application data files, technical report, etc.).

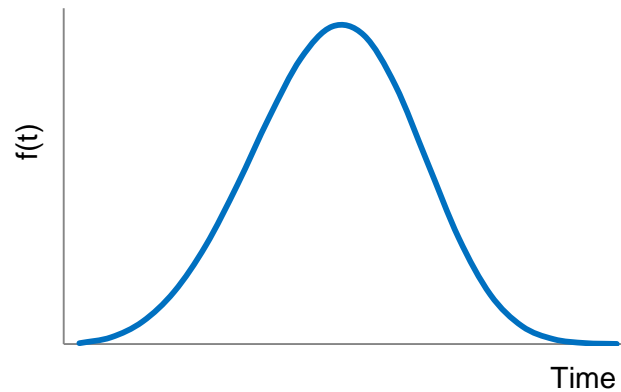
#### 3.1 STATISTICAL BACKGROUND

Reliability can be defined as “*the probability that an item will perform its intended function without failure for a specified period of time*”. Reliability, defined as a probability, is, therefore, quantified using the mathematics of probability and statistics. The reliability characteristics of an item can be described using four inter-related mathematical functions:

- a) Probability density function (pdf) =  $f(t)$
- b) Cumulative distribution function (cdf) = unreliability function =  $F(t)$
- c) Reliability function =  $R(t)$
- d) Hazard rate (or failure rate) =  $\lambda(t)$

The probability density function describes the distribution of values as a function of a specific variable (e.g. time) and can be represented mathematically or on a graphical plot (where the x-axis represents time). Figure 1 shows an example of a probability density function.

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**Figure 1: Probability density function**

The relationships between the four mathematical functions are as follows:

$$R(t) = 1 - F(t)$$

$$\lambda(t) = \frac{f(t)}{R(t)}$$

Other parameters, often used in reliability engineering, include Mean Time to Failure (applicable to non-repairable items) and Mean Time Between Failure (applicable to repairable items).

Note: MTTF (or MTBF) is frequently used as an indicator of “average life” of an item, which may be completely incorrect. The exponential distribution describes the situation where the hazard (or failure) rate is constant. It can be shown that the mean value of the exponential distribution (i.e. MTTF (or MTBF)) is  $1/\lambda$ , and that 63.2% of items will have failed by  $t = \text{MTTF}$  (or MTBF).

The most frequently used distributions in reliability engineering include the normal (or Gaussian) distribution, exponential distribution and Weibull distribution.

The well-known bathtub curve is useful to indicate that the hazard (or failure) rate can be decreasing, constant or increasing over time. It consists of the infant mortality, useful life and wear-out life parts, as shown in Figure 2. Failures occurring during infant mortality (i.e. early failures) are typically caused by workmanship problems (i.e. quality control related), failures occurring during useful life (i.e. random failures) are typically externally induced and failures occurring during wear-out (i.e. end of life failures) are typically caused by fatigue and wear. Although mathematically identical, hazard rate is used for non-repairable items, while failure rate is used for repairable items.

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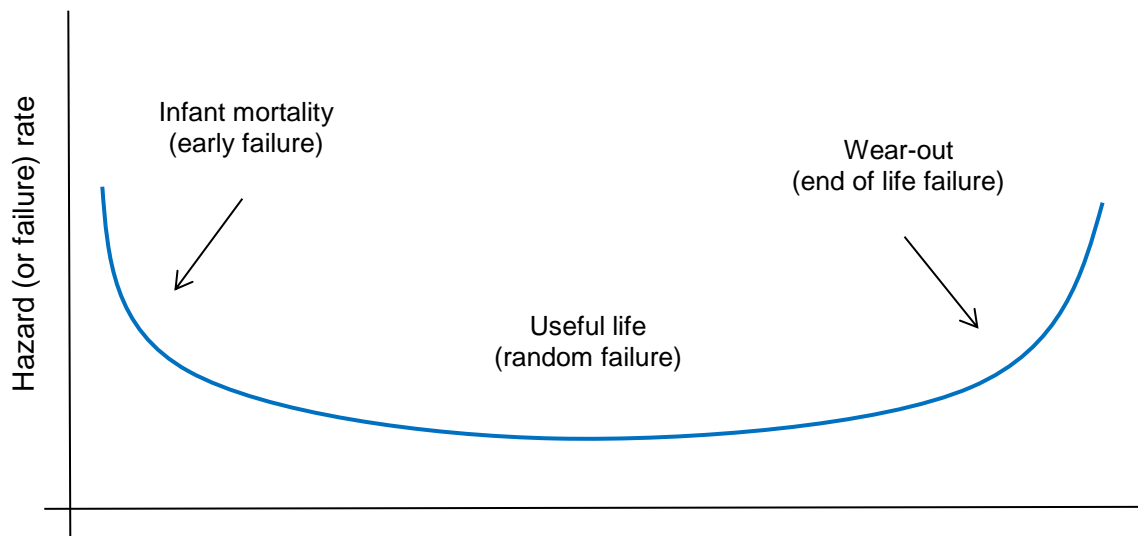


Figure 2: Bathtub curve

### Censored data

Complete life data means that the time-to-failure of all units in a sample is known. In many practical cases, all units in a sample may not have failed or the exact times-to-failure of all the units are not known. This type of data is called censored data and it may be right censored (also called suspended data), interval censored or left censored. Right censored data, as shown in figure 3, is frequently found in failure data. Unless it is correctly analysed, it will obviously result in incorrect and misleading results. Censored data is typically handled by all LDA software applications.

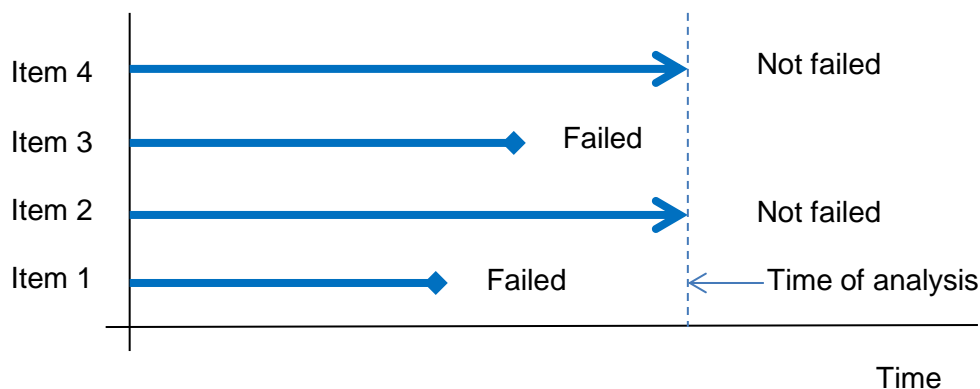


Figure 3: Right censored data

### Confidence bounds

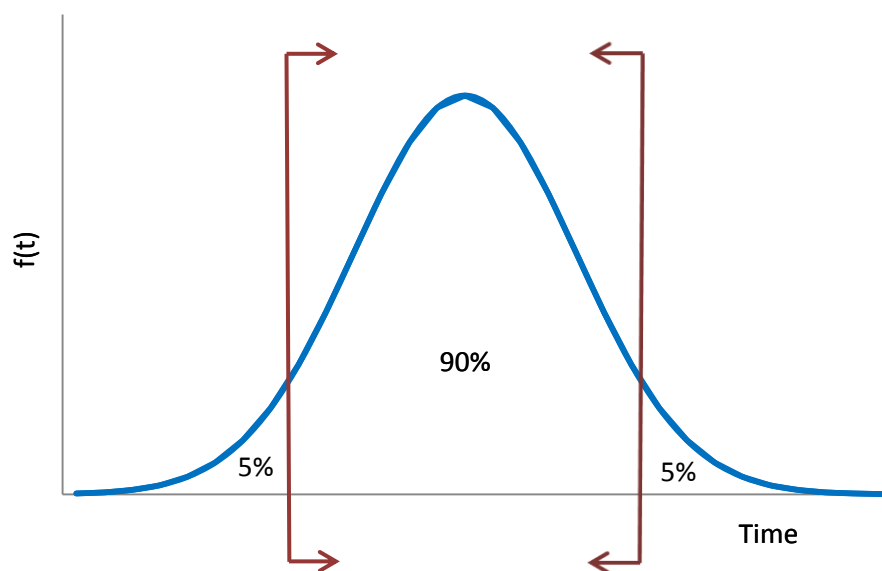
Since LDA results are estimates based on the observed lifetimes of a sample of units, there is uncertainty in the results due to the limited sample sizes. Confidence bounds (also called confidence intervals) are used to quantify this uncertainty due to sampling error by expressing the confidence that a specific interval contains the quantity of interest. It is not impossible to know the exact value of a reliability parameter, unless failure data for every single unit in the population can be analysed (which is usually an unrealistic situation). Confidence bounds define the range within which the specific value is likely to occur within a certain percentage of the time. It should be noted that each value is an estimate of the true value (which is an unknown). Confidence bounds can be expressed as two-sided or one-sided bounds and are typically calculated by all LDA software applications.

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Two-sided confidence bounds define a closed interval where a certain percentage of the population is likely to lie. For example, Figure 4 shows 90% two-sided confidence bounds, where 90% of the population lies between X and Y with 5% less than X and 5% greater than Y.



**Figure 4: Two-sided confidence bounds**

Essentially, one-sided confidence bounds are an open-ended version of two-sided bounds. A one-sided bound defines the point where a certain percentage of the population is either higher or lower than the defined point. This means that there are two types of one-sided bounds, namely upper and lower. An upper one-sided bound defines a point that a certain percentage of the population is less than. Conversely, a lower one-sided bound defines a point that a specified percentage of the population is greater than. For example, if X is a 95% lower one-sided bound, this would indicate that 95% of the population is greater than X (as shown in Figure 5). If X is a 95% upper one-sided bound, this would indicate that 95% of the population is less than X (as shown in Figure 6).

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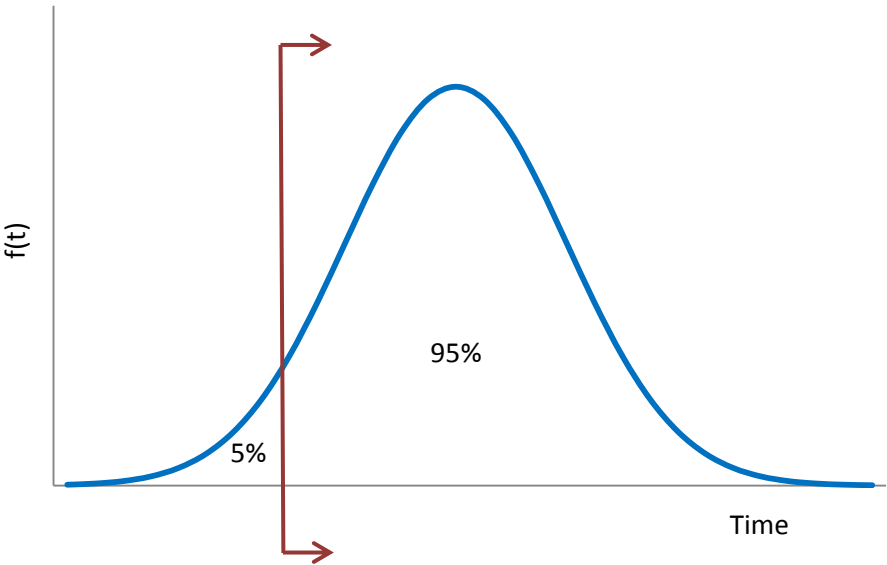


Figure 5: Lower one-sided confidence bound

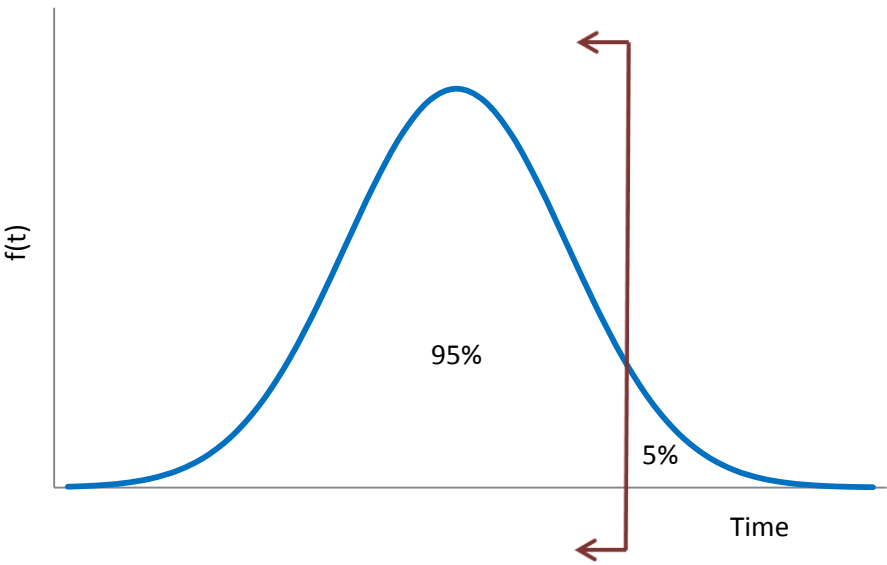


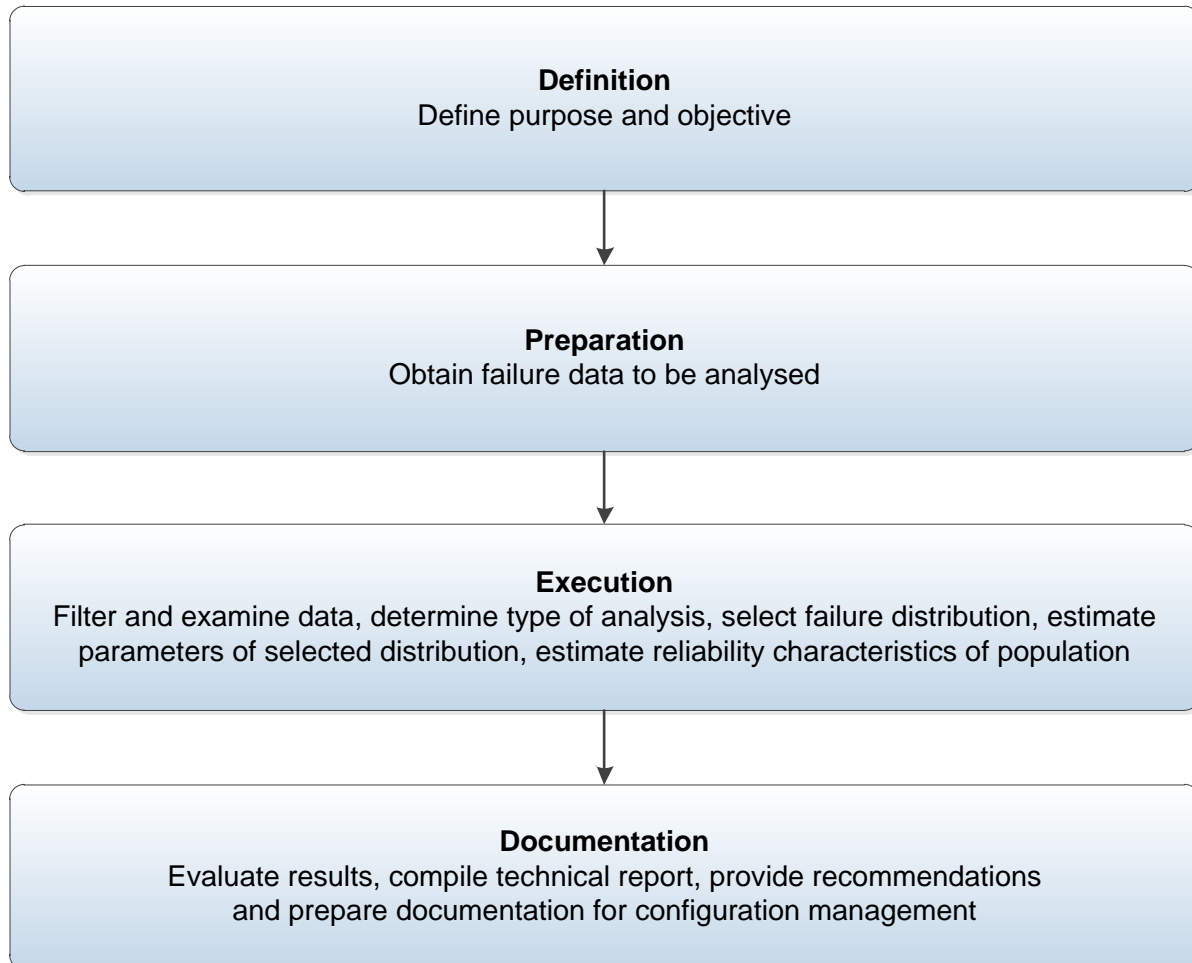
Figure 6: Upper one-sided confidence bound

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### 3.2 LIFE DATA ANALYSIS PROCESS

LDA can be used to estimate the life of all items in a population by determining the parameters of a statistical distribution derived from failure data of a representative sample of units. LDA consists of the following four basic sequential steps:

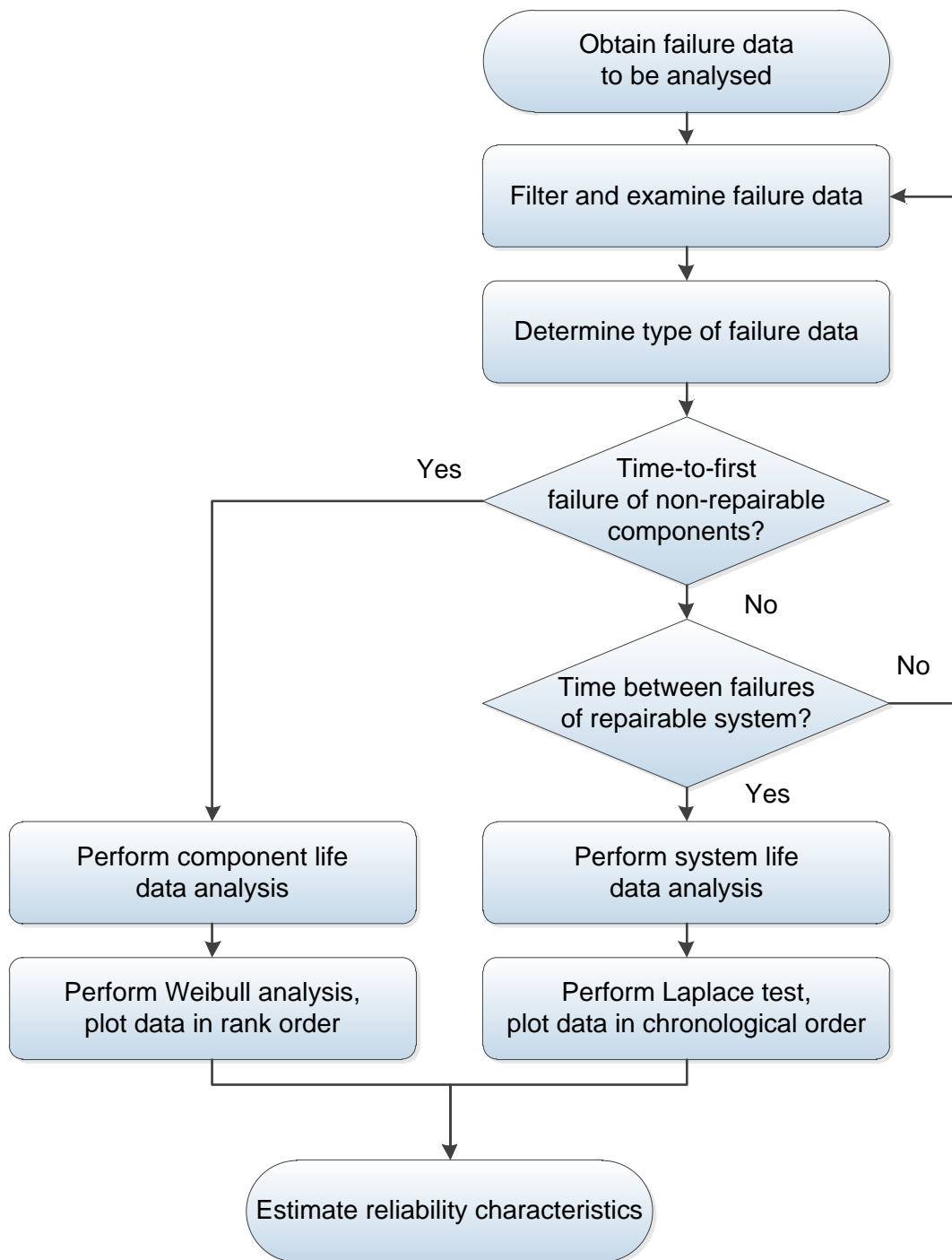


**Figure 7: Life data analysis basic steps**

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The LDA execution sequence diagram can be seen in Figure 8.



**Figure 8: Life data analysis execution sequence diagram**

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### 3.3 COMPONENT LIFE DATA ANALYSIS

LDA should be performed using an applicable software application but can also be performed using graphical methods, i.e. probability plotting. Since knowledge of graphical methods is beneficial to understand the results obtained from a software application, an introduction on probability plotting is included in this section.

Weibull analysis is widely used in LDA due to the flexibility of the Weibull distribution (i.e. it can be used to approximate many other distributions), the easy interpretation of distribution parameters and their relation to the different parts of the bathtub curve.

#### 3.3.1 Two parameter Weibull distribution

The reliability function for the two parameter Weibull distribution is:

$$R(t) = e^{-(t/\eta)^\beta}$$

where  $t$  = time

$\beta$  = shape parameter

$\eta$  = characteristic life = scale parameter  
(or life at which 63.2% of population will have failed)

The probability density function  $f(t)$ , reliability function  $R(t)$  and hazard (or failure) rate  $\lambda(t)$  for different values of  $\beta$  is shown in Figure 9.

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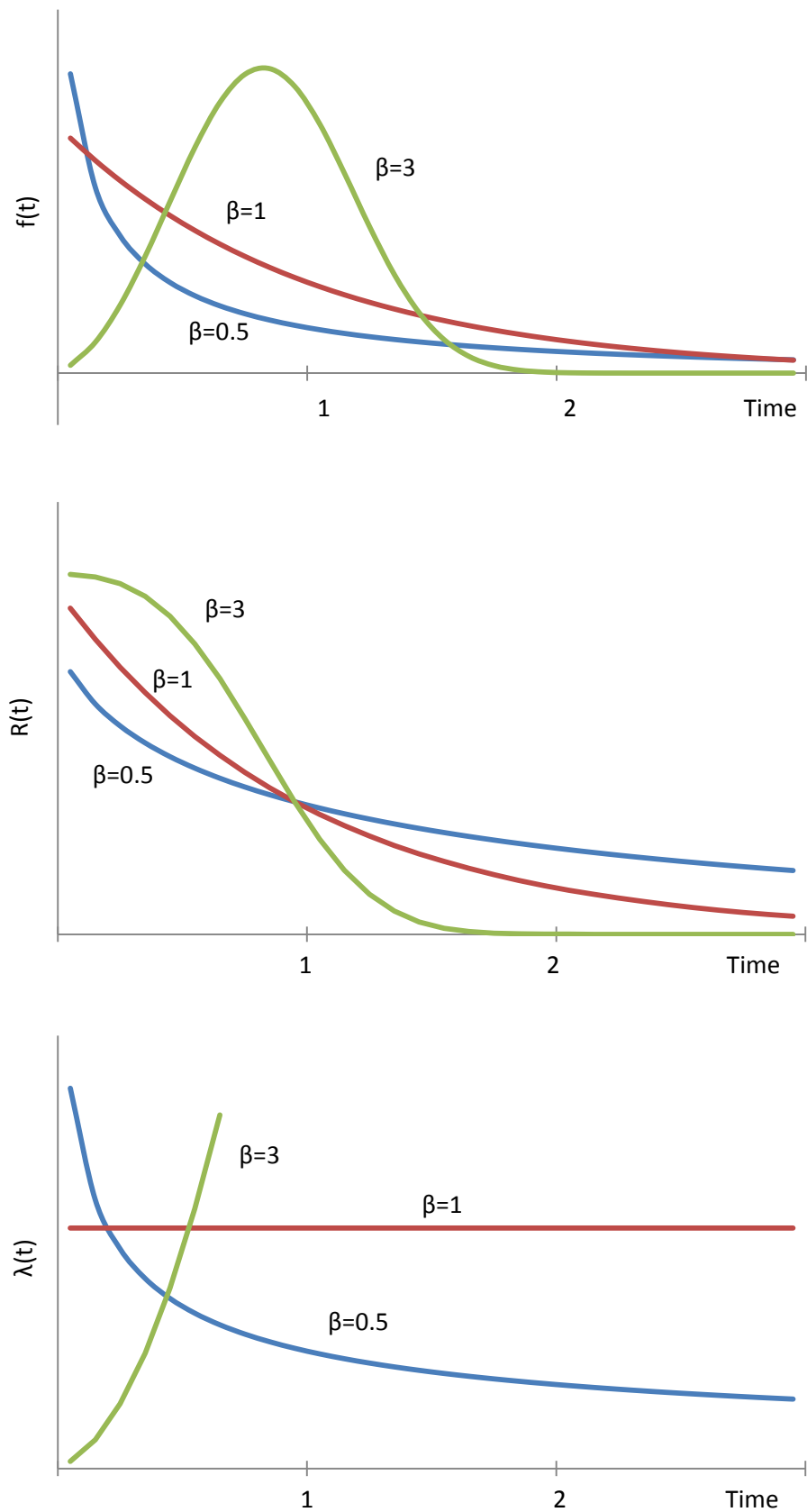


Figure 9: Two parameter Weibull distribution (for different values of  $\beta$ )

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If the data can be modelled using the two parameter Weibull distribution, the reliability function can be transformed to plot as a straight line on specially prepared Weibull plotting paper. An example of Weibull probability paper is shown in Figure 10. The y-axis represents unreliability  $F(t) = 1 - R(t)$  and the x-axis represents time (or other usage parameter). Given the x and y-value for each data point, all values can easily be plotted and a best-fitting straight line can be drawn through the data points. Parameter  $\beta$  can be determined as the slope of the line (it is not physically calculated, but traced on the paper) and parameter  $\eta$  can be determined as the time corresponding to 63.2% unreliability on the y-axis.

Another parameter that is often used to specify or measure reliability is the B-life, which is the time by which a certain percent of the population can be expected to fail. A typical value is 10% (e.g. B<sub>10</sub> life of 15 years is the same as 90% reliability for 15 year mission life). B-life can, therefore, easily be obtained from a Weibull analysis.

Since data analysis is typically performed on a sample of the population, and to allow for the fact that each failure represents a point on a distribution, ranking of data is used to improve the accuracy of estimation. More commonly, mean ranks are used to plot symmetrical distributions, such as the normal distribution. Median ranking is the method most frequently used in probability plotting, particularly if the data is known not to be normally distributed. An example will illustrate the applicability of the mean rank in section 3.3.4

The following methods can be used to perform mean and median ranking:

$$\text{Mean rank } r_i = \frac{i}{N+1}$$

$$\text{Median rank } r_i = \frac{i-0.3}{N+0.4}$$

where  $i$  = failure order number

$N$  = sample size

Table 1 shows mean and median ranks for a sample size of 5 items.

**Table 1: Mean and median ranking**

Failure order number	No rank	Mean rank	Median rank
1	20%	16.67%	12.96%
2	40%	33.33%	31.48%
3	60%	50.00%	50.00%
4	80%	66.67%	68.52%
5	100%	83.33%	87.04%

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### 3.3.2 Three parameter Weibull distribution

The reliability function for the three parameter Weibull distribution is:

$$R(t) = e^{-(t-\gamma/\eta)^\beta}$$

where  $t$  = time

$\beta$  = shape parameter

$\eta$  = characteristic life = scale parameter

(or life at which 63.2% of population will have failed)

$\gamma$  = location parameter = minimum life = failure free time

Therefore, the three parameter Weibull distribution is useful to analyse failure data where failures only start after a finite time. Since the three parameter Weibull plot cannot be represented by a straight line on a Weibull plot, it necessitates the use of LDA software applications.

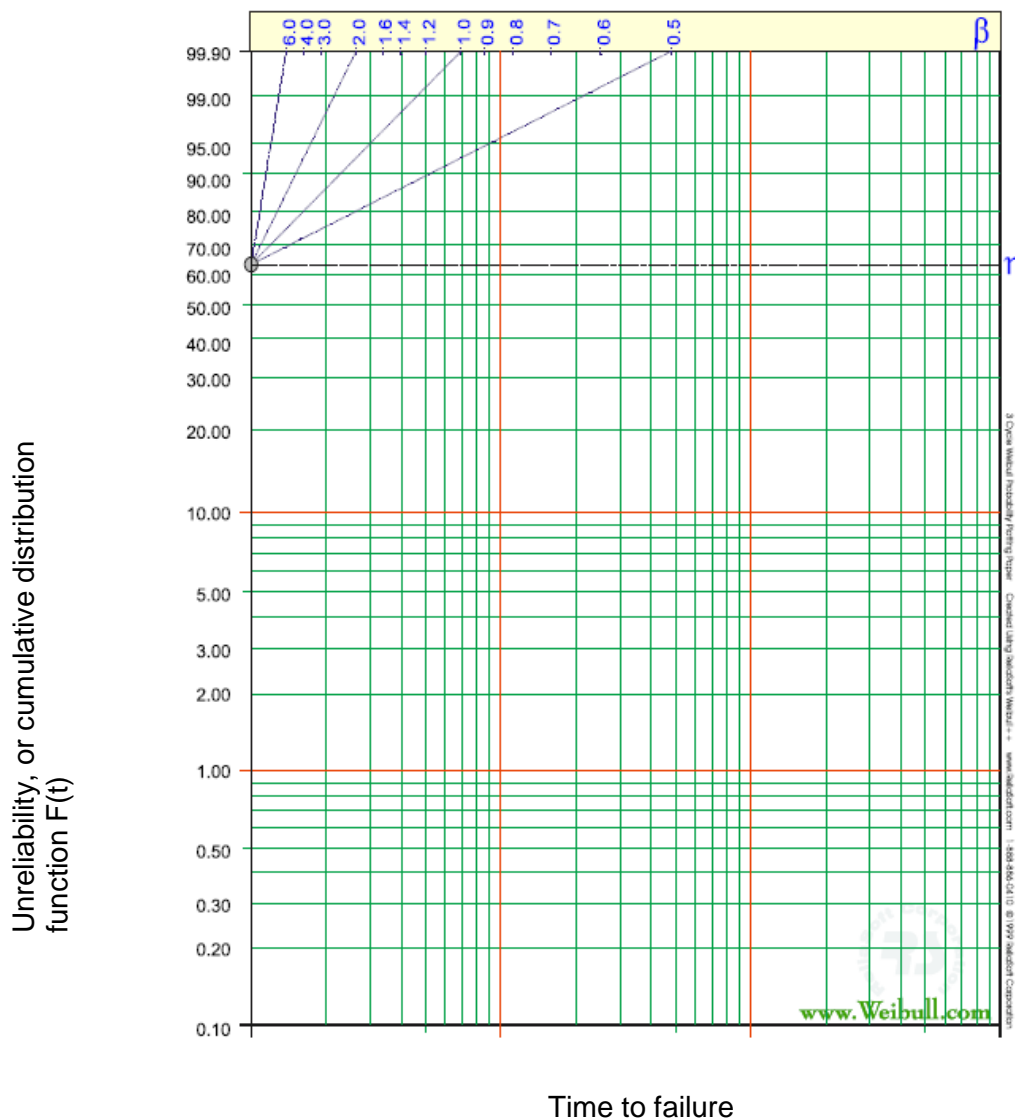


Figure 10: Weibull probability paper

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### 3.3.3 Relationship of shape parameter with bathtub curve

The shape parameter  $\beta$  determines the shape of the Weibull distribution, which enables the Weibull distribution to approximate a number of other failure distributions. Therefore, it can also be used to identify a specific part of the bathtub curve, as shown in Figure 11:

#### $\beta < 1$

- decreasing failure rate
- infant mortality (or early failures) part of bathtub curve
- workmanship or quality control related failures

#### $\beta \approx 1$

- constant hazard (or failure) rate
- useful life part of bathtub curve
- random failures or externally induced failures

#### $\beta > 1$

- increasing failure rate
- wear-out part of bathtub curve
- end-of-life failures

$\beta = 1$ , therefore, implies an exponential distribution and  $\beta \approx 3$  approximates the normal (or Gaussian) distribution.  $\beta$  values larger than 6 are not uncommon (and reflects accelerated or fast wear-out) but should be treated with suspicion (depending on the expected failure mechanism).

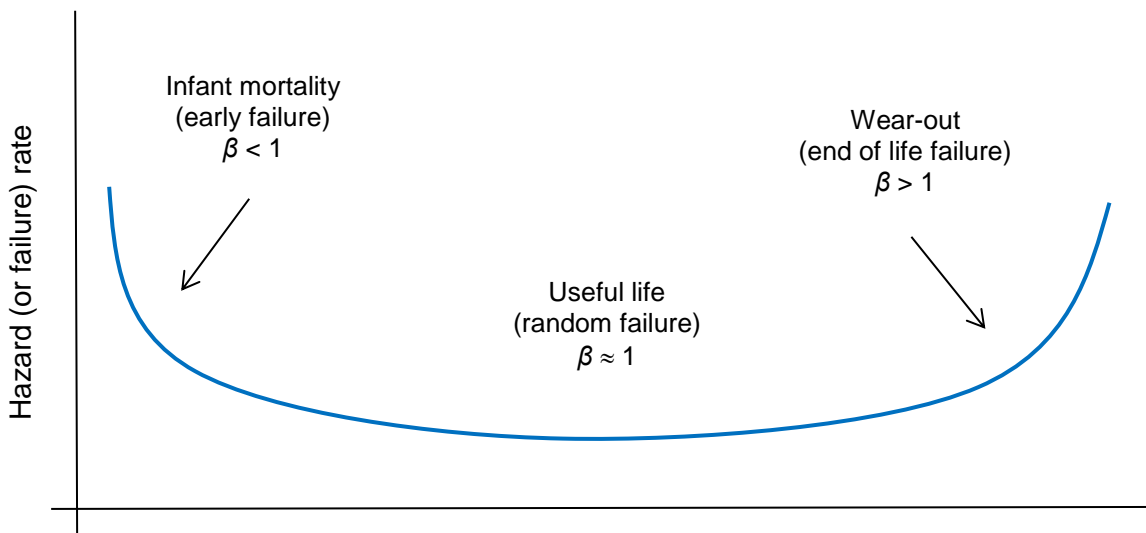


Figure 11: Relationship between bathtub curve and Weibull shape parameter

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### 3.3.4 EXAMPLE 1

Determine the reliability function for a component for which the following field failure data is available (i.e. individual time-to-failure for each part):

**Table 2: Time-to-failure of individual items**

Item number	Time-to-failure (h)
1	300
2	200
3	350
4	100
5	250
6	450
7	150
8	500
9	400

As shown in Table 3, the failure data should be ordered (i.e. sorted according to time-to-failure) and the median ranks determined.

**Table 3: Ordered time-to-failure data with median ranks**

Order number	Time-to-failure (h)	Median rank (sample size = 9)
1	100	7%
2	150	18%
3	200	29%
4	250	39%
5	300	50%
6	350	61%
7	400	71%
8	450	82%
9	500	93%

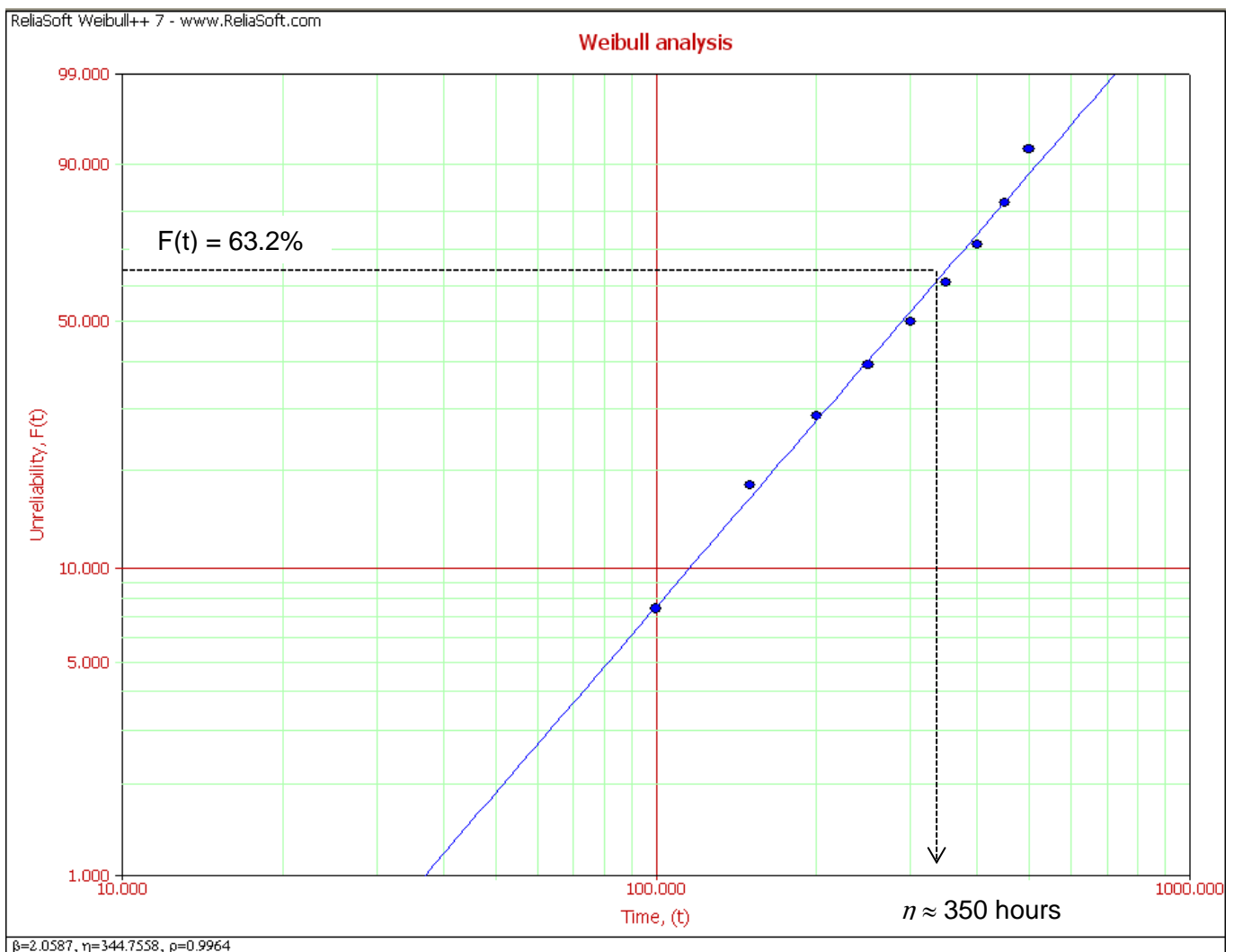
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These data points (i.e. time-to-failure vs. median rank) are shown in Figure 12. The distribution parameters are  $\beta = 2.06$  and  $\eta = 344.76$  hours (the gradient of the best fit line can be used as indicated at the top of Figure 10, to determine the beta parameter). With  $\beta \approx 2$  and  $\eta \approx 350$ , the reliability at any time can thus be calculated using the following equation:

$$R(t) = e^{-(t/350)^{2.0}}$$

For example, reliability for a mission time of 100 hours is  $R(100) = e^{-(100/350)^{2.0}} = 0.92$  (or 92%).



**Figure 12: Weibull analysis**

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### 3.4 SYSTEM LIFE DATA ANALYSIS

#### 3.4.1 Stochastic point processes

Situations in which discrete events occur randomly in a continuum (e.g. time) cannot be represented by a single continuous distribution function. Failures occurring in repairable systems (i.e. series of discrete events) is an example of a *stochastic point process*, which can be analysed using the statistics of *event series*.

The Poisson distribution function describes the situation in which events occur randomly and at a constant average rate. This situation is described by a *homogenous Poisson process* (HPP). An HPP is a *stationary* process, since the distribution of the number of events in an interval of fixed length does not vary, regardless of when the interval is sampled.

The Poisson distribution function is given by:

$$f(x) = \frac{(\lambda x)^n}{n!} \exp(-\lambda x) \quad \text{for } n = 0, 1, 2, \dots$$

where  $\lambda$  is the mean rate of occurrence and  $\lambda x$  is the expected number of events in  $(0, x)$ .

In a *non-homogenous Poisson process* (NHPP), the point process is non-stationary (rate of occurrence is a function of time) so that the distribution of the number of events in an interval of fixed length changes as  $x$  increases. Typically, the discrete events (e.g. failures) might occur at an increasing or decreasing rate.

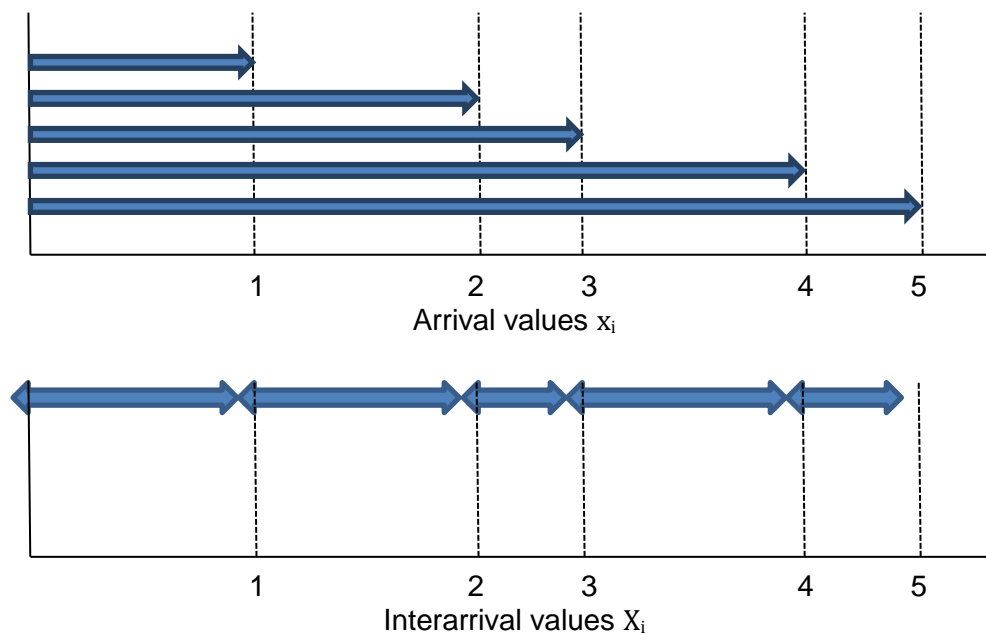
An essential condition of any homogenous Poisson process is that the probabilities of events occurring in any period are independent of what has occurred in preceding periods. An HPP describes a sequence of independently and identically exponentially distributed (IIED) random variables. A NHPP describes a sequence of random variables, which are neither independent nor identically distributed.

#### 3.4.2 Laplace test

When analysing data from a stochastic point process, it is important to determine whether or not the process has a trend, specifically, to know whether a failure rate is increasing, decreasing or constant. This can be done by analysing the *arrival values* of the event series. The arrival values:  $x_1, x_2, \dots, x_n$  are the values of the independent variables (e.g. time) from  $x = 0$  at which each event occurs. The *interarrival values*:  $X_1, X_2, \dots, X_n$  are the intervals between successive events 1, 2, ..., n, from  $x = 0$ . The distinction between arrival and interarrival values can be seen in Figure 13.

If  $x_0$  is the period of observation, then the test statistic for trend is the following:

$$U = \frac{\sum x_i / n - x_0 / 2}{x_0 \sqrt{1/12n}}$$



**Figure 13: Arrival and interarrival values**

This is called the centroid test or the Laplace test and it compares the centroid of the observed arrival values with the mid-point of the period of observation. The Laplace test is one method to determine whether or not discrete events in a process have a trend:

- If  $U = 0$ , there is no trend  
(i.e. process is stationary)
- If  $U < 0$ , the trend is decreasing  
(i.e. interarrival values tend to become progressively larger)
- If  $U > 0$ , the trend is increasing  
(i.e. interarrival values tend to become progressively smaller)

If the period of observation ends at an event,  $(n-1)$  is used instead of  $n$ , and the time to the last event is excluded from the summation  $\sum x_i$ . When the Laplace value is greater than (less than)  $+1.96$  ( $-1.96$ ), it can be stated with at least 95% confidence (considering a two sided confidence interval) that there is a significant trend upward (downward).

The centroid test is theoretically adequate if  $n \geq 4$ , when the observation interval ends with an event and if  $n \geq 3$  when the interval is terminated at a predetermined time. The method is also called *time series analysis*.

The Laplace test is a non-parametric analysis method and cannot quantify a trend, i.e. it can only identify a trend. If the actual rate of change in the trend is required, a parametric analysis method should be used, e.g. Crow (AMSAA) reliability growth model.

### CONTROLLED DISCLOSURE

**3.4.3 EXAMPLE 2**

Suppose the following interarrival times (in hours) were collected for a repairable system over a 3,800 hour period:

- a) 1,600, 800, 400 and 200 hours
- b) 400, 1,600, 200 and 800 hours
- c) 200, 400, 800 and 1,600 hours

These interarrival times can be converted to arrival times (i.e. start from the same point in time) and the Laplace value can be calculated. Table 4 shows arrival times for the first data set.

**Table 4: Arrival and interarrival times**

Interarrival times $X_i$	Arrival times $x_i$
1,600	1,600
800	2,400
400	2,800
200	3,000

$$U = \frac{\sum x_i / n - x_0 / 2}{x_0 \sqrt{1/12n}}$$

$$U = \frac{9,800/4 - 3,800/2}{3,800 \sqrt{1/12 * 4}} = 1.0028$$

Therefore:

- a) For 1,600, 800, 400 and 200 hours,  $U = +1.00 \Rightarrow$  increasing failure rate
- b) For 400, 1,600, 200 and 800 hours,  $U = 0.00 \Rightarrow$  constant failure rate
- c) For 200, 400, 800 and 1,600 hours,  $U = -1.09 \Rightarrow$  decreasing failure rate

The “mean” interarrival time of each data set is 750 hours, yet the patterns of failure of the data sets are completely different.

Reference: TC Adams, *The Laplace Test*,

<http://kscsma.ksc.nasa.gov/Reliability/Documents/Laplace%20Test.pdf>

**CONTROLLED DISCLOSURE**

**3.4.4 EXAMPLE 3**

Determine if the following failure data represents a trend (i.e. is the failure rate increasing, constant or decreasing) for the following arrival values  $x_i$  and interarrival values  $X_i$  between 12 successive failures (observation ends at the last failure).

**Table 5: Ordered arrival times**

Interarrival times $X_i$	Arrival times $x_i$
175	175
21	196
108	304
111	415
89	504
12	516
102	618
23	641
38	679
47	726
14	740
51	791

$$U = \frac{\Sigma x_i / n - x_0 / 2}{x_0 \sqrt{1/12n}}$$

$$U = \frac{5,514/11 - 791/2}{791 \sqrt{1/(12 \times 11)}} = 1.54$$

Therefore, the interarrival times are becoming shorter, i.e. the failure rate is increasing. The existence of a trend in the data indicates that the interarrival values are not independently and identically distributed (IID). This is an important point to consider in the analysis of failure data.

**CONTROLLED DISCLOSURE**

### 3.4.5 EXAMPLE 4

For repairable systems, the distribution of times to first failures is far less important than the *failure rate* or *rate of occurrence of failures* of the system. Any repairable system may be considered as an assembly of parts, the parts being replaced when they fail. The system can be thought of as comprising of “sockets” into which non-repairable parts are fitted. As each part fails, a new part takes its place in the “socket”. Reliability of repairable systems is concerned with the pattern of successive failures of the “sockets”.

Consider the data used in Example 3. The interarrival and (chronological ordered) arrival values between successive part failures are shown in columns 1 and 2 of Table 6.

**Table 6: Chronological ordered data**

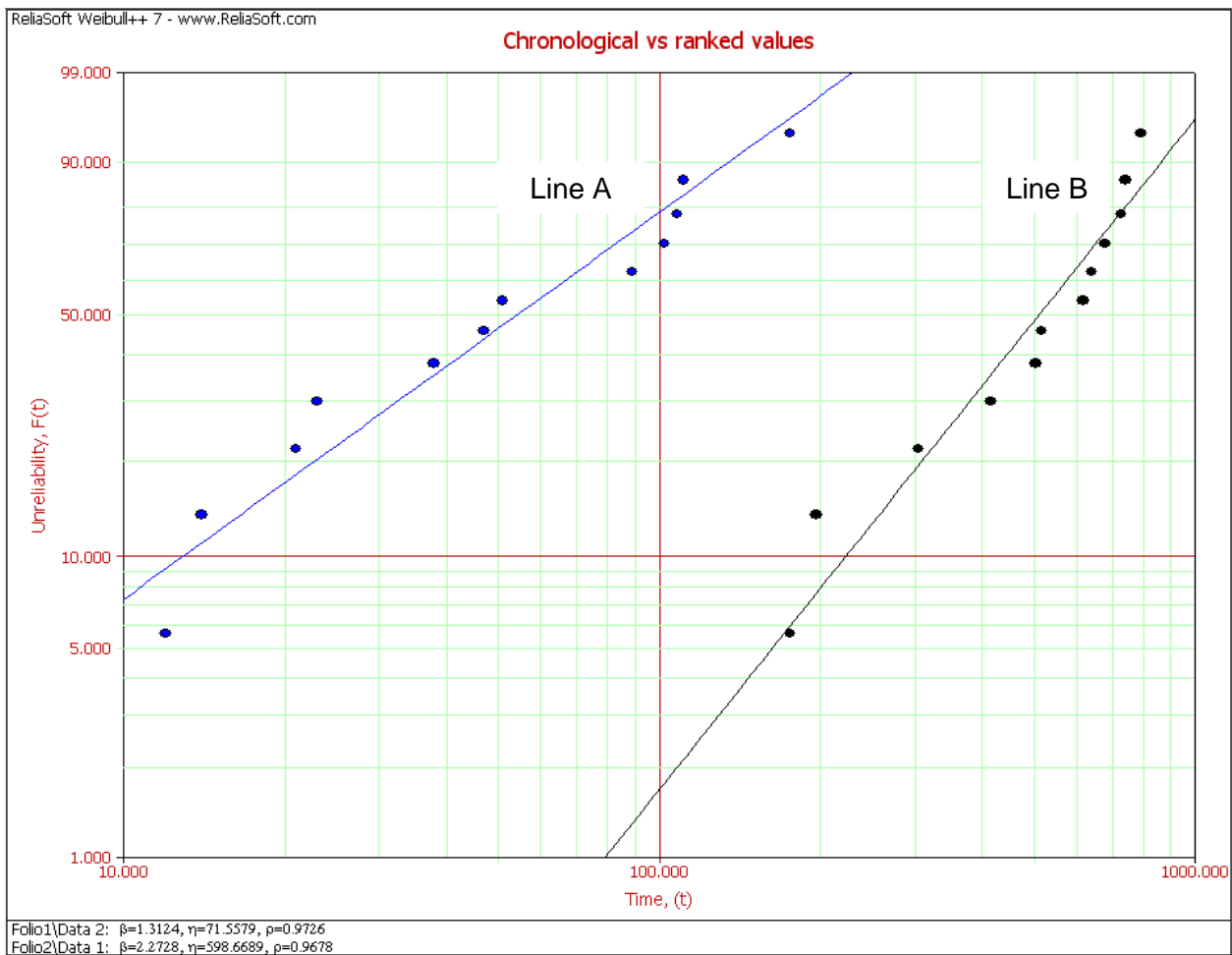
1	2	3
$X_i$	Chronological $x_i$	Ranked $x_i$
175	175	12
21	196	14
108	304	21
111	415	23
89	504	38
12	516	47
102	618	51
23	641	89
38	679	102
47	726	108
14	740	111
51	791	175

Example 3 showed that the failure rate is increasing, since the interarrival values tend to become shorter (i.e. the interarrival values are not IID). If the centroid test was not performed, the data might be ordered using rank order and analysed using Weibull analysis. The results are shown as Line A on Figure 14 and show an apparent exponential life distribution. This is obviously a misleading result, since there is clearly an increasing failure rate trend for the “socket” when the data is studied chronologically.

System reliability over a period of time can be derived by plotting the cumulative times to failure in chronological order (column 2) rather than in rank order. This is shown as Line B in Figure 14. It is evident that Line A and Line B show different results in terms of both the shape parameter  $\beta$  and the scale parameter  $\eta$ .

### CONTROLLED DISCLOSURE





**Figure 14: Weibull plot for repairable system**

This example shows how important it is for failure data to be analysed correctly, depending on whether the analysis is concerned with the reliability of a non-repairable part or with a system consisting of “sockets” into which parts are fitted. A typical system, consisting of several parts that exhibit independent failure patterns, can be analysed in a similar manner (i.e. multisocket systems).

If part times to failure (in a series system) are independently and identically exponentially distributed (IID exponential), the system will have a constant failure rate, which will be the sum of the reciprocals of the part mean times to failure:

$$\lambda_s = \sum_{i=1}^n \frac{1}{x_i}$$

The assumption of IID exponential for parts times to failure within their sockets in a repairable system can be misleading. Refer to O’Connor, page 342 for more details on possible reasons.

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The following people were involved in the development of this document:

- Reliability and Safety Engineering during Design Work Group

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